The return of the Frisbee

Aerodynamic effects on Frisbee flight

Oded Rose

Abstract: Most people who play with Frisbees have probably noticed a rather unusual phenomenon. When the Frisbee is thrown with a steep angle of inclination, it will, at some point, make a rather sharp turn and descend while coming back to the thrower. This phenomenon was apparently not discussed in scientific literature. This article compares the Frisbee with another returning object, the boomerang, and outlines the differences between the two returns. A wind tunnel study of the Frisbee was performed to determine its aerodynamic properties. The article then discusses the various factors affecting the Frisbee flight and shows, using numerical analysis, how each in its turn affects the returning capabilities of the Frisbee.

INTRODUCTION

How many objects can you think of which come back after throwing them into the air with an initial forward motion ? Not many, and that is obviously because some force is needed to act on this object and deviate it from its original path and reverse its direction of motion. An external force such as a head wind could do the job given a strong enough wind and a low mass/area ratio of the object. However, this is of less interest to us than such objects as boomerangs and Frisbees which return of their own accord.

As a physics student at the Technion, Israel's Institute of Technology, it was one of my tasks to design and carry out a lab project. As I had already done quite a bit of lab work, I was looking for something I could do outdoors, most preferably at the beach, a place where Physics students rarely visit. I went to the beach thinking of some sort of a device which would measure the force of the waves so I could correlate between the wave force and height. While I enjoyed riding some good surf I noticed a boy who was playing with a Frisbee. What caught my attention was the fact that the boy was playing on his own.

Frisbee is a popular outdoor game. You can play it with two or more people by throwing it from one to the other with a low flat inclination while giving it a spin using a quick wrist motion. When properly thrown, the Frisbee floats gracefully through the air until it slowly descends into the other player's hands. Experienced Frisbeeiers use numerous tricks when playing. such as: letting the arriving Frisbee spin on one finger before catching it. Stretching out the arms to allow the arriving Frisbee touch one palm, role along the arm and chest or neck and onto the other arm before catching it or even allowing it to continue to the next player.

Playing Frisbee on your own requires a slightly different throwing technique if you don't want to run around and fetch it. One does not throw it at a low flat inclination but rather one releases the Frisbee at a steeper angle. After its release the Frisbee will rise to a certain point while moving forward. Then it will start dropping and if it was properly thrown it will stop its forward motion and also reverse. The complete path will look to someone watching from the side as following an upsidedown 'U' which is also tilted diagonally. It also seems that the Frisbee itself retains its angle and orientation relative to the ground.

What causes the Frisbee to stop its forward motion and reverse ? One of the most common answers I got from people with whom I discussed this phenomenon was "Well, isn't it just like a boomerang ?". This was my initial belief as well. I started out by making an extensive search for bibliography related to the subject matter but I could not find anything published on this specific Frisbee phenomenon. After some more investigation on the physics behind the returning boomerang it was clear that there are major differences between the two.

As an anecdote, except for being a great sports game, the only other practical use for a Frisbee I have found reference to, was a recent study of the inverted Frisbee as a dust deposition gauge.[1] The inverted Frisbee proved to be three and four times better in its collection efficiency of dust particles, than the standard gauges.

THE BOOMERANG'S MOTION

Several papers describing the physics behind the boomerang's return have been published even as early as 1898. A comprehensive study, made by Felix Hess,[2] was published in the November 1968 issue of the Scientific American. The study included a computer assisted analysis of the forces that affect the flight path of the boomerang. It provided plots which very much resembled the photographs of actual boomerang flights taken at night using a small light bulb embedded in the boomerang.

It is a common belief that the "returning boomerang" originated in Australia for recreation purposes. However, there is some evidence that similar toys were developed independently in other parts of the world. The original boomerangs were probably the hunting or war boomerangs that were designed to hit accurately and fly straight. Aerodynamically they are more complicated than the returning boomerangs. The classic returning types have a banana or a 'V'-like shape. However, this 'V' shape has hardly anything to do with the boomerang's ability to return. A boomerang with three, four or even eight arms may return just as well. The boomerang is thrown with its arms initially rotating around its center of mass in an almost vertical plane. A small tilt to the right (for right handed boomerangs) is required as will be explained shortly.

The arm's cross section is that of an airfoil. When the arm moves through air, this shape forces the air on the convex side of the arm to move faster than a parallel section of air on the flat side, because of the greater distance it travels. Due to Bernulie's effect a lower pressure on the convex side is thus created. The net force on the arm will point from the flat to the rounded side (the same effect makes an airplane wing work). When the boomerang moves through air the arm which points upwards has a velocity as a result of the boomerang's rotation which is in the same direction as the forward velocity of the boomerang. The velocity of the arm pointing downward opposes the forward velocity of the boomerang and hence the upper arm "feels" the air going faster than does the lower arm. Therefore the aerodynamic force on the upper arm will be greater then the lower one; this difference creates a torque around an axis which coincides with the forward velocity vector.

The rotating boomerang, however, does not give way to the torque, which to a viewer from behind, seems to "want" to turn the boomerang counter-clockwise. Rather, the boomerang changes its direction around a vertical axis. This motion is called percession. It is similar to a fast motorcycle or bicycle which turns left or right when its rider leans to either side. This process continues and if the boomerang is initially given enough forward velocity, it will complete a full circle and with a bit of luck or experience, land at the thrower's feet. The small tilt which is initially given to the boomerang compensates for gravity's effects by slightly pointing the lift force vector upwards. In fact, the axis of the boomerang's rotation changes during flight from being almost horizontal to vertical owing to what is known as the boomerang's "lying down effect".

THE FRISBEE'S MOTION

The Frisbee, unlike the boomerang with its initial horizontal axis of rotation, normally has a vertical axis of rotation with a slight tilt backwards. As mentioned before, during most of its flight the Frisbee retains its orientation and relative angle to the ground while in the air. This is due to its rotation that stabilizes the Frisbee in a similar way to a gyroscope. A further discussion on the effects of rotation on the Frisbee's motion will be made later in this paper.

The Frisbee's overall cross-section is similar to an airfoil (see fig. 1) and principally works in the same way as outlined previously. As a result of its motion through air, the induced lift force counter acts gravity and reduces the down pulling force. This allows the Frisbee to remain for a longer time in the air than another object which does not have these aerodynamic capabilities. The

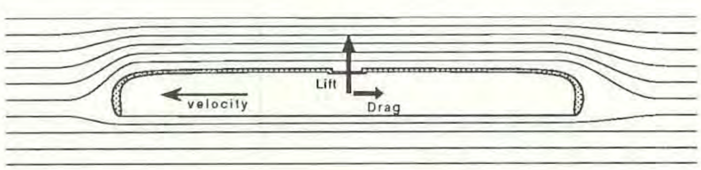

Fig. 1. A Frisbee cross-section with air stream lines illustrating its airfoil features.

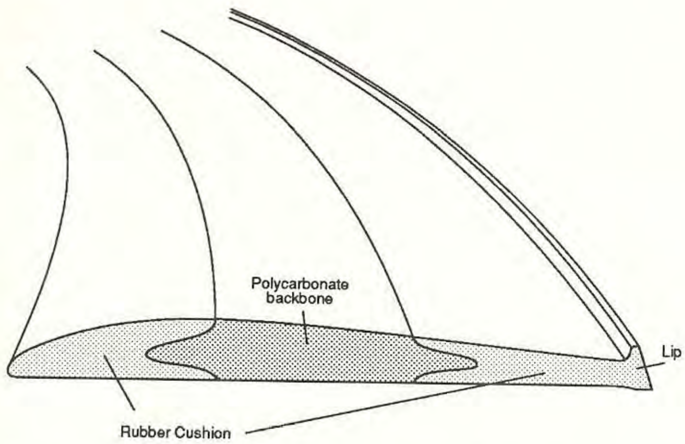

Fig. 2. A cross section of the Aerobie showing its low profile which results in a low aerodynamic drag and the spoiler lip that causes its center of lift to move slightly backwards to the center of rotation. These are the main reasons for the Aerobie's achievement of the Guiness record distance for a thrown heavier than air object.

feeling we get when looking at its flight, is that the Frisbee gracefully floats on a cushion of air.

When throwing the Frisbee at a low angle of inclination it behaves quite "normally" during most of its flight time. At a certain point, however, after slowing down quite considerably, the Frisbee seems to stop in mid air and slowly descend. This becomes even more peculiar when the angle of inclination is increased to somewhere around 20° (depending on the particular Frisbee). At some point the Frisbee not only stops its forward motion but also, in what seems to counter the laws of physics, reverses and while already descending returns towards the thrower.

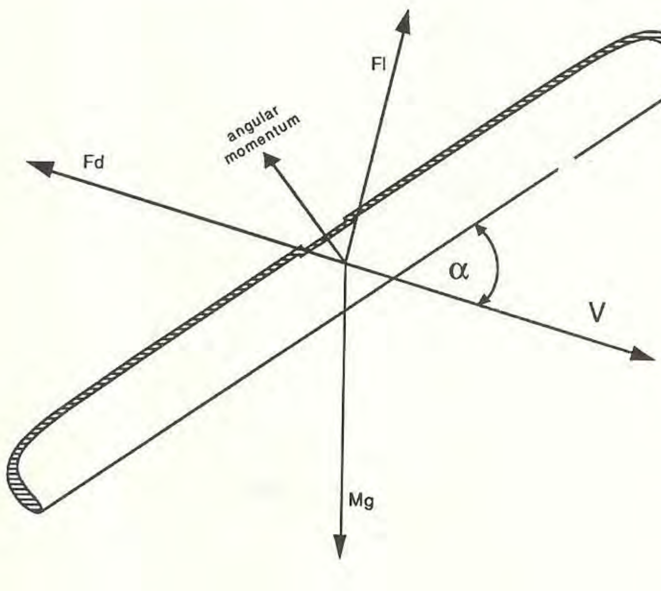

Fig. 3. At any angle of attack (α), Drag (Fd) points 180° to the Frisbee's velocity (V) and lift (Fl) is perpendicular to them. Fl and the angular momentum both lie in a plane which is perpendicular to the plane of the Frisbee. At any given angle of attack Fl and Fd are dependent on V and are proportional to its square while gravity (Mg) is practically constant.

One of the more recent versions of the Frisbee, the Aerobie, is in the Guiness book of records for holding the longest flight by a thrown, heavier-than-air object, 1046 ft. Although this is more than twice the Frisbee's 456 ft record, the Aerobie does not have the extraordinary feature of returning. The Aerobie was invented in the early 80's by Alan Adler a lecturer in electrical engineering at Stanford University who used a computer to design it. He tried several designs before arriving at the slick flat ring equipped with a spoiler lip on the outer rim (see fig. 2). The spoiler lip reduces the lift-slope of the Aerobie's leading half while increasing the lift-slope of the trailing half. This moves the center of lift slightly back to the center of rotation (and gravity). This is practically the only way to make a spinning object fly straight. Add to this the Aerobie's very low aerodynamic drag and you get the reasons for the object's ability to fly over great distances.

FACTORS INVOLVED IN THE FRISBEE'S MOTION

Three factors affect the Frisbee's flight path after launching. These are: Gravity (g), Aerodynamic lift (F_L) and Aerodynamic drag (F_d). Drag force always points 180° to the Frisbee's velocity vector while lift is always perpendicular to it (see fig. 3). The drag and lift forces are usually represented as follows:

$$F_{drag} = {}^1/_2\, C_d \rho A v^2$$
$$F_{lift} = {}^1/_2\, C_L \rho A v^2 \qquad (1)$$

C_d and C_L are dimensionless drag and lift coefficients largely determined by the object's shape and cross-section but strongly dependent on the angle of attack (α) which is the angle between the plane of the Frisbee and its velocity vector 'v'. ρ is the air density and 'A' is the maximum cross-sectional area of the object. Acceleration due to aerodynamic forces is:

$$a = -{}^1/_2\, \rho A v^2 (C_d{}^2 + C_L{}^2)^{1/2}/M \qquad (2)$$

If wind exists it could affect the Frisbee's motion quite drastically owing to the Frisbee's low mass/area ratio.

Throwing a Frisbee even in a moderate wind, could become a difficult task and probably not worth-while doing. If we compare the Frisbee and the boomerang ability to fly in wind, the latter, if thrown into the wind, will fly and return successfully. The increased aerodynamic lift (caused by the head wind) will force the boomerang to percess faster. It will simply make a smaller circle plus it will probably not complete the circle but rather get dragged backwards forming a 'C' shape path.

The velocity of a head or tail wind (v_w) could easily be added to the Frisbee's ground velocity (v_f) and we could then discuss the Frisbee's relative air velocity (v_{rel}) where, $v_{\text{rel}} = v_f - v_w$. Cliff Frohlich showed, using numerical analysis, that throwing a discus into a head wind of up to 20 m/sec, actually increases the distance achieved by the discus by several meters.[3] Wind effects on the Frisbee could be a subject for a further investigation. In this study I assume, for simplification purposes, no wind and a flight path in two dimensions only. The Frisbee's flight path is usually not only in a two dimensional vertical plane, but if we assume it were, the equations of motion would be much more practical to deal with while still being very instructive.

The top surface of Frisbees is very often covered with thin grooves or ridges. Counter to our intuition, the roughening of the top surface actually decreases the drag on the Frisbee. This is because the rough surface creates a turbulent boundary layer that results in a narrower wake of low pressure behind the Frisbee. It has also been reported,[4] that the rough surface contributes to a higher lift slope. At least with boomerangs, rough surface ones returned quicker than those with smooth surface due probably to the higher lift on the arms. The wind tunnel experiments of this paper used a smooth top Frisbee.

EQUATIONS OF MOTION

Based on the above assumptions, the equations of motion that were used for this study were:

$$X'' = -{}^1/_2\,(\rho A v^2/M)(C_d \cos\beta - C_L \sin\beta),$$
$$Y'' = -g + {}^1/_2\,(\rho A v^2/M)(C_L \cos\beta - C_d \sin\beta). \qquad (3)$$

β is the angle between the horizontal plane and v. These equations can be solved if the initial conditions are known. The initial conditions include the initial release velocity v_0, the release angle β_0 (the angle between the horizontal plane and v_0) and the release height y_0. At any particular time T its velocity is defined as:

$$X' = v_{x0} - {}^1/_2\,(\rho A/M) \times \int_0^T (C_d \cos\beta + C_L \sin\beta)dt,$$
$$Y' = v_{y0} - gT + {}^1/_2\,(\rho A/M) \times \int_0^T (C_L \cos\beta - C_d \sin\beta)dt.$$

Hence

$$X = v_{x0}T - {}^1/_2\,(\rho A/M) \times \int_0^T\!\!\int_0^t v^2(C_d \cos\beta + C_L \sin\beta)dt'dt,$$
$$Y = -{}^1/_2 gT^2 + v_{y0}T + {}^1/_2\,(\rho A/M) \times \int_0^T\!\!\int_0^t v^2(C_d \cos\beta - C_L \sin\beta)dt'dt.$$

For numerical calculations it is much easier to integrate over small increments of time Δt. Our calculations would improve if we used X''' and Y''' when $\beta = \tan^{-1}(v_y/v_x)$

$$X''' = (\rho A/2M)\{2(v_x x'' + v_y y'')(-Cd \cos\beta - C_L \sin\beta) + (v_x y'' - v_y x'')[(\delta C_d/\delta\alpha - C_L)\cos\beta + (\delta C_L/\delta\alpha - C_d)\sin\beta]\},$$
$$Y''' = (\rho A/2M)\{2(v_x x'' + v_y y'')(C_L \cos\beta - C_d \sin\beta) + (v_x y'' - v_y x'')[(-\delta C_L/\delta\alpha - C_d)\cos\beta + (\delta C_d/\delta\alpha - C_L)\sin\beta]\}.$$

and for each time increment Δt

$$\Delta X = X'\Delta t + {}^1/_2 X''\,\Delta t^2 + {}^1/_6 X'''\Delta t^3$$
$$\Delta Y = Y\Delta t + {}^1/_2 Y''\,\Delta t^2 + {}^1/_6 Y'''\Delta t^3$$

Experimenting with several Δt's yielded that increments of 0.1 sec were the most practical to use.

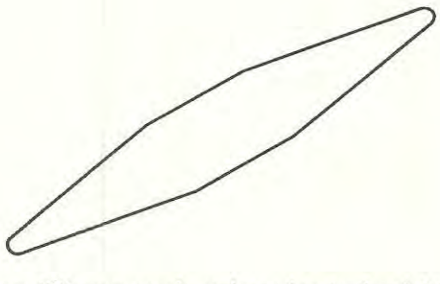

Fig. 4. A discus tilting upwards and moving to the right experiences higher lift and drag forces on its leading half than on its trailing half due to the leading half's bigger angle of attack. This results in a torque which makes the discus roll sideways. The Frisbee is relatively free from such a torque.

EFFECTS OF ROTATION ON THE FRISBEE'S MOTION

As mentioned above, the most important effect of the Frisbee's rotation is to stabilize it during flight. The Frisbees total energy is: $E_{tot} = E_k + E_p + E_r$; where E_k is the Firsbee's kinetic energy, E_p is its potential energy and E_r is its rotational energy. During a normal flight the combined magnitude of the kinetic and potential energy is about ten times larger, on average, than its rotational energy. This was another consideration for not including rotation calculations in this study. However, the aerodynamic forces combine with the Frisbee's angular momentum to apply torques that, although relatively small, are not negligible. A Frisbee with an angular momentum vector pointing downward would experience a larger Lift force on its left side (viewed from behind) due to this side's slightly higher air velocity. The Lift force is effectively applied to the left of the Frisbee's center of mass (and rotation). This creates a torque vector pointing forward causing the front edge of the Frisbee to tilt (pitch) upwards. This motion is similar to the above described percession of the boomerang.

The discus' rate of percession is $1^1/_2$°/sec. Field tests did not reveal this information about the the Frisbee but

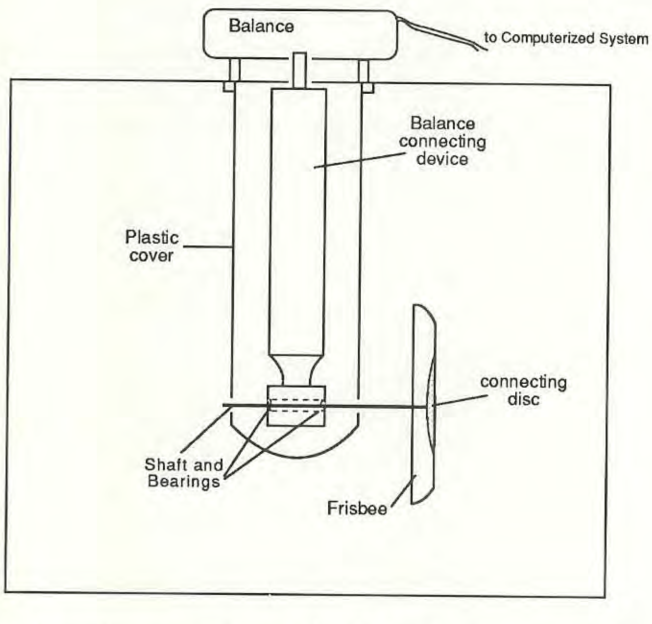

Fig. 5. A wind tunnel cross section showing the first experiment setup. The plastic Frisbee was connected to an adhesive plastic disc which was screwed onto the shaft. The plastic cover eliminated wind effects on the connecting device. Additional calculations would have been needed to get accurate results because the readings were not directly from the Frisbee but rather from the effects of the Frisbee on the connecting device. The unsuccessful experiment made these calculations unnecessary.

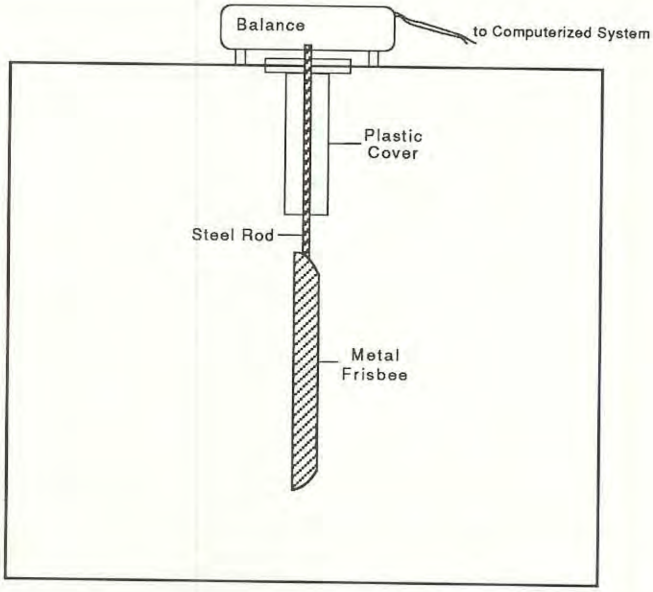

Fig. 6. A wind tunnel cross section showing the metal frisbee experiment setup. This device proved to be solid and gave stable results with low error readings for several wind velocities and all angles of attack. The plastic cover greatly reduces wind effects on the steel rod.

it seems that it is in the same magnitude as the discus' despite the Frisbee's smaller moment of inertia. The discus also experiences higher lift and drag forces on its leading half due to its cross sectional shape (see fig. 4). This creates another torque which forces the discus to roll sideways. The Frisbee's relatively straight bottom line frees it from this effect.

DESCRIPTION OF THE EXPERIMENTS

The concept behind the experiments was to verify the validity of the above theory compared to field test results. Two sets of experiments were designed. The first was an outdoor study of the Frisbee's flight path and the second was a set of wind tunnel experiments to reveal the Frisbee's aerodynamic characters.

Field tests:

Different possibilities were investigated, including a visit to the well equipped "Hogla" (rock-partridge - Hebrew) sports shooting club. Their clay-pigeon launching devices with some minor modifications could have been very suitable for this purpose. However, their limited angle of inclination (0°-24°) prevented their usage. Various ideas for making launching devices and using different methods of illuminating the Frisbee were examined. Finally a 50 frames/sec video camera was used which recorded throwing the Frisbee at various initial angles and velocities.

This experiment consisted of two parts. The first was to try and correlate between the initial parameters of velocity, angle and height of release to the final distance achieved by the Frisbee. In this part the camera was kept still with its frame capturing the thrower together with several meters above and in front of him. Using basic geometry the tapes were analyzed frame by frame, obtaining the initial velocity, release angle and height. Release velocities varied between 11-19 m/sec. By pre-marking one side of the Frisbee with a bright color it was possible to determine the average rate of its initial spin, 6-7 turns/second. In the second part, the camera was moved further away so it could follow the entire flight path of the Frisbee. This information gave a clear and more detailed picture but still, more of a qualitative observation of the different types of trajectories. The inconsistency of the releasing parameters (v_0 and y_0) prevented a more precise analytical study.

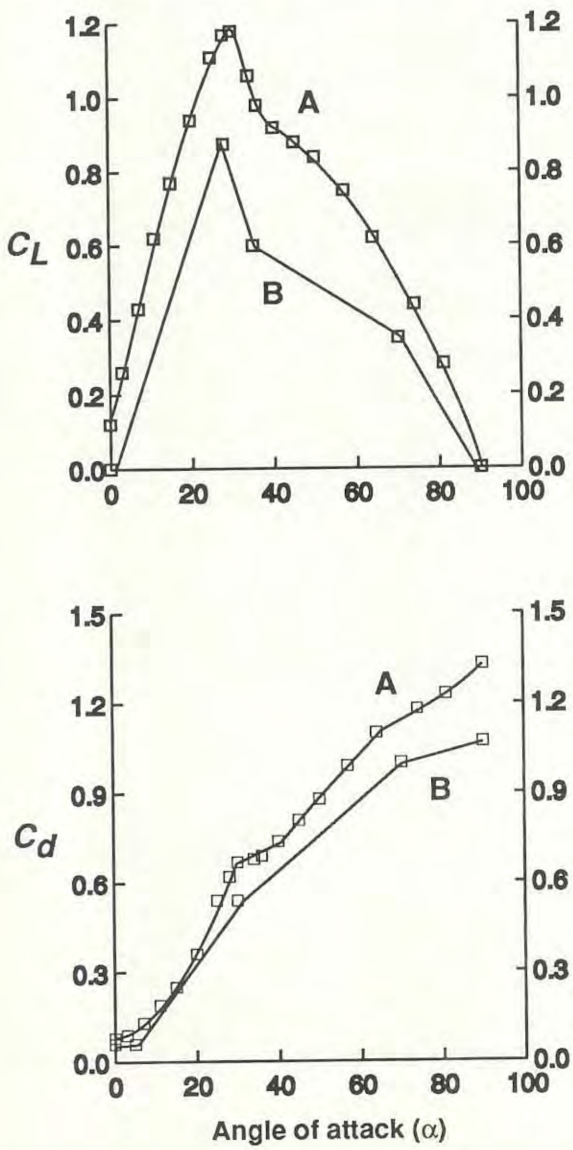

Fig. 7. Lift and drag coefficients: A - Frisbee. Measured at the metal Frisbee wind tunnel experiment. These measurements are for wind velocity of 14.1 m/sec. B - Discus. These values were used by Cliff Frohlich in his study of the Aerodynamic effects on the discus.

Wind tunnel experiments:

The main objectives of this experiment were to determine the Frisbee's lift and drag coefficients (C_d and C_L). These are usually not much affected by the wind velocities with which we work (10-25 m/sec), but are crucially dependent on the object's angle of attack.

These experiments were conducted at the Faculty of Aeronautics "Dantziger" no. 1 wind tunnel. It is a relatively small, open-circuit blower type tunnel. Its working section is 8 meters long, 80 cm wide and 65 cm in height. The roof-top balance is read by a computerized system. Three types of forces are measured, lift, drag and torque. The first two were outlined above. Torque is much smaller in magnitude compared with them and was not measured in this experiment. The forces are coupled which means that each one affects the other two. If we measure these forces directly we will receive incorrect results. The computerized system helps by reading the balance calibration results and automatically deriving a transformation matrix. This matrix later on filters the experiment results and displays correct readings for the forces and coefficients.

Several stages went by before a device that was reliable and stable at all angles of attack was created. The first one was designed to enable testing the effects of spin on the Frisbee's aerodynamic properties (see fig. 5). The results from testing this device were unusable due to the results' very large standard of deviation. This was later explained by the fact that the Frisbee was not stable and wobbled around its connection point which seriously affected the readings. The final design was a much simpler device which although could not spin, proved to be very stable at all angles of attack. It was a one to one steel model of the Frisbee, carefully spun to the correct measurements and welded onto a 10 mm diameter steel rod (See fig. 6).

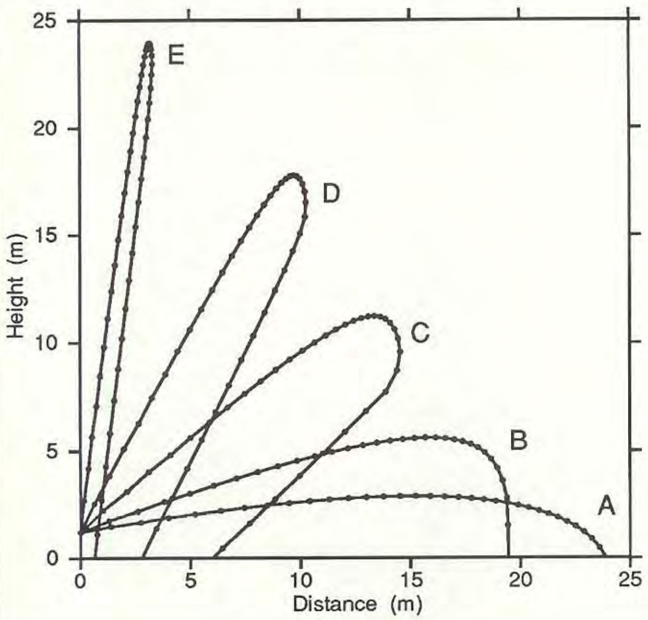

Fig. 8. Trajectories of Frisbee thrown with various release angles (β_0): A - 10°, B - 20°, C - 40°, D - 60°, E - 80°. Small black circles indicate the Frisbee's position every 0.1 sec. Note that the larger is β_0, the less ground distance is covered even at angles below 45°. The release velocity (v_0) is 15 m/sec, release height (h_0) is 1.2 m. A = 0.42 m², M = 0.1 kg, ρ = 1.22 kg/m³ and g = 9.8 m/sec².

The steel Frisbee measurements were:
Diameter - 2.3 x 10^{-1} m,
Height - 2.7 x 10^{-2} m.

Air density (ρ) was 1.226 kg/m³. Lift and drag forces and coefficients were measured at three wind velocities: 14.1, 17.4 and 20.2 m/sec. After plotting the results of C_d and C_L for the different angles of attack (0°-90°) at these velocities and making the appropriate error calculations, I found that the three graphs varied only slightly among themselves.

Wind tunnel results:

The results presented in table I and fig. 7 were measured at wind velocity of 14.1 m/sec. A study of the angles of attack between 90°-180° revealed an almost mirror image of these results. Hence for $0° < \alpha < 90°$:

$C_d(\alpha) \cong C_d(180° - \alpha)$,
$C_L(\alpha) \cong - C_L(180° - \alpha)$.

C_L readings for angles over 90° are negative because the Lift force vector points to the opposite direction.

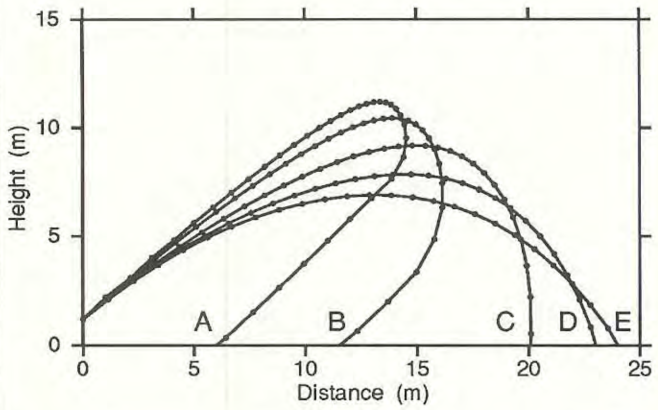

Fig. 9. Trajectories of theoretical Frisbees with various masses: A - 100 g, B - 200 g, C - 500 g, D - 1 kg, E - 2 kg. β_0 = 40°, v_0, y_0, ρ, g and A are as in fig. 8.

RESULTS OF NUMERICAL CALCULATIONS

To write a practical computer program which could make the flight simulations, a number of simplification assumptions were made. Mentioned above were wind velocity set to zero and a two dimensional plane of motion perpendicular to the plane of the ground. The small torque force, discussed above, was not included in these calculations. I also assumed that the Frisbee rotates only about an axis perpendicular to its face (i.e. no "wobble"), and that the plane of the Frisbee intersects with the plane of the ground along a line perpendicular to the direction of the throw (i.e. the Frisbee tilts neither to the right or the left).

With these assumptions equation (3) applies and seven variables remain: β_0, ρ, g, v_0, y_0, M and A. The latter two were also examined as variables since Frisbees come in a variety of sizes and masses.

Effect of β_0

Following the example of the boy's Frisbee on the beach, I have since thrown Frisbees numerous times at various angles of release. When doing that, one notices right away that a steeper angle of release results in a smaller ground distance covered. From a certain angle and above the Frisbee will also return. The numerical calculations confirm this phenomenon (see fig. 8). With ρ, v_0, y_0, M and A as in fig. 8, I found out that a maximum distance would be achieved with the Frisbee released at a

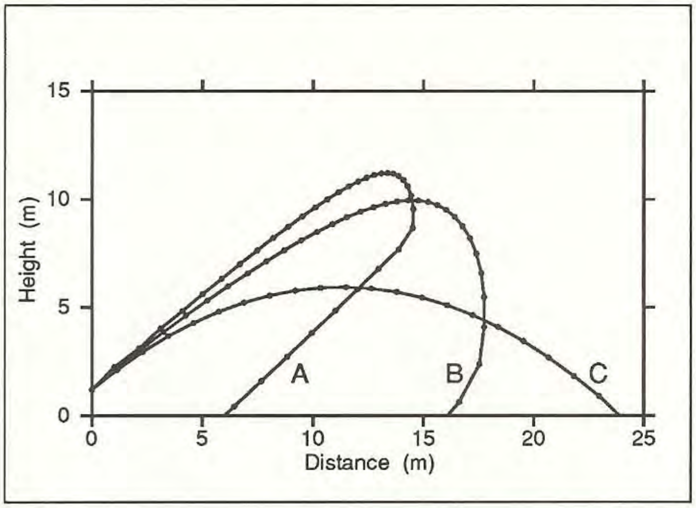

Fig. 10. Trajectories of Frisbees with various air densities (ρ): A - 1.22 kg/m³, B - 0.4 kg/m³, C - no air (vacuum). $\beta_0 = 40°$, v_0, y_0, g, M and A are as in fig. 8.

7° angle. This is contrary to the textbook figure of 45° which could apply for a ball and quite different from the Discus' optimal release angle of 37° (for no wind)[3]. Launching the Frisbee with a release angle of 21° and more, results in the Frisbee's return (see fig. 8). Following my assumption that the Frisbee does not change its orientation in the air during flight I kept β_0 as the constant relative angle between the plane of the Frisbee and the plane of the ground. Because of the throwing

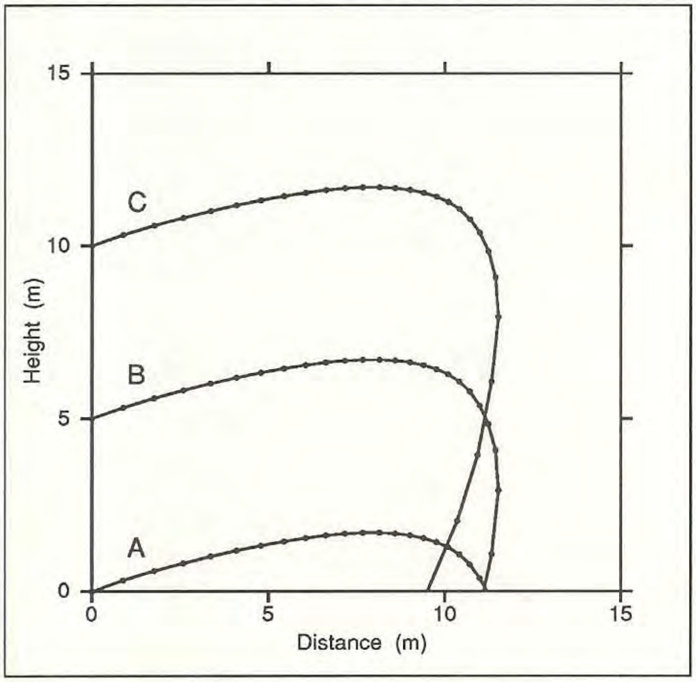

Fig. 11. Frisbee trajectories at various release heights (y_0): A - 0 m, B - 5 m, C - 10 m. Note that the path of A is common to all three trajectories. Here $\beta_0 = 20°$, v_0, = 10 m/sec, ρ, g, M and A are as in fig. 8.

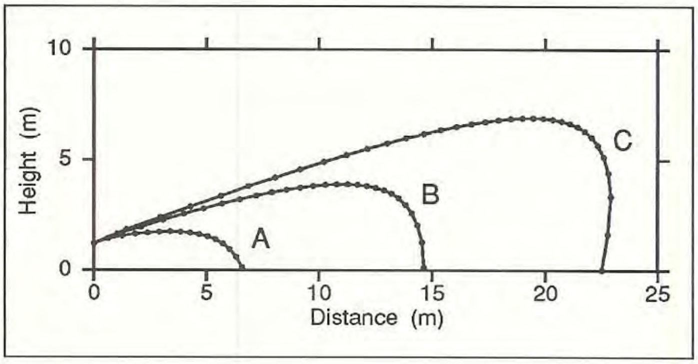

Fig. 12. The effect of various release velocities (v0) is mainly on the height achieved before vx reaches zero. A - 7 m/sec, B - 12 m/sec, C - 17 m/sec. $\beta_0 = 40°$, ρ, y_0, g, M and A are as in fig. 8.

technique of the Frisbee, v_0 coincides with the plane of the Frisbee and thus $\alpha_0 = 0$.

Effect of *M* and *A*

Unlike the discus for example, where its area (A) and mass (M) are fixed by the sport authorities, Frisbees come in various sizes and masses. the small collection of Frisbees that I have, varies between one of 94 mm in diameter and its mass is about 35 g, and one which is 245 mm in diameter and weighs 150 g. The field tests and most of the computer simulations were done with a 100 g Frisbee that measured 230 mm across (A = 0.042 m²). This one seemed to have very good returning capabilities and the calculations confirmed this observation. From eq. (2) the aerodynamic acceleration is inversely proportional to the mass and directly proportional to the area. In the case of β_0 = 40°, increasing M results in less of returning capability and a greater distance achieved (see fig. 9). The opposite pattern would follow when A is increased. Although I have not made simulations varying A, from eq. (2) a larger area would result in greater aerodynamic force, and thus, increase the Frisbee's ability to return. Launching the Frisbee with β_0 = 20° shows a slightly different pattern. The 1kg theoretical Frisbee made the longest flight. It went 40% higher and 2 m further than the 2 kg theoretical Frisbee. The others (0.1, 0.2 and 0.5 kg) achieved greater heights but made less ground distance.

Effect of ρ and g

Air density ρ can vary nearly 50% between a high-temperature, high-altitude site and a low-temperature low altitude one. Fig. 10 illustrates the effect of various air densities on the Frisbee's trajectory. In vacuum the Frisbee follows a parabolic path. Increasing the air's density "brings back" the Frisbee's returning capabilities. The denser is the air, the sharper is the Frisbee's turn and the closer it comes to the releasing point.

Changing altitude above sea level also results in a very small change in gravity. In practice g varies no more than 0.5% (about 5 cm/sec^2) over the surface of the earth. At the poles g is larger than at the equator. These variations would have only very little effect on the Frisbee's trajectory and therefore I have not made the specific calculations. Eq. (3) predicts that increasing g reduces the distance covered by the Frisbee because of an earlier ground hit. Its returning capabilities would also be slightly reduced because of the smaller a/g ratio.

Effect of y_0 and v_0

One of the initially less expected findings that resulted from these calculations came when I varied the release height y_0 . The calculations showed that even with a small release angle such as 20° (see fig. 11), and given a high enough release point the Frisbee will return. This was later confirmed by throwing the Frisbee out of windows of several buildings. Even a very heavy Frisbee may return. To make the 2 kg "Frisbee" return, y_0 needs to be increased much more. At y_0=40 m, β_0=20° and ρ, v_0 and A as in fig. 8 the 2 kg "Frisbee" returned 1.35 m. what is important is that the Frisbee would be above ground when its horizontal velocity v_x reaches zero. From this time and onwards the distance that the Frisbee will return is obviously dependent on the height above ground but it depends as before on β_0, ρ, M and A.

The major effect of the release velocity v_0, is, if we are interested in the returning phenomenon, to bring the Frisbee high enough before it begins its descend. fig. 12 shows different trajectory patterns given a change in v_0.

DISCUSSION AND CONCLUSIONS

why does the Frisbee return

Aerodynamic forces are a very significant factor in the Frisbee's flight. At a release velocity of 15 m/sec the 100 grams Frisbee experiences a ratio of aerodynamic (a) to gravitational (g) forces which approaches unity; a/g = 0.85.

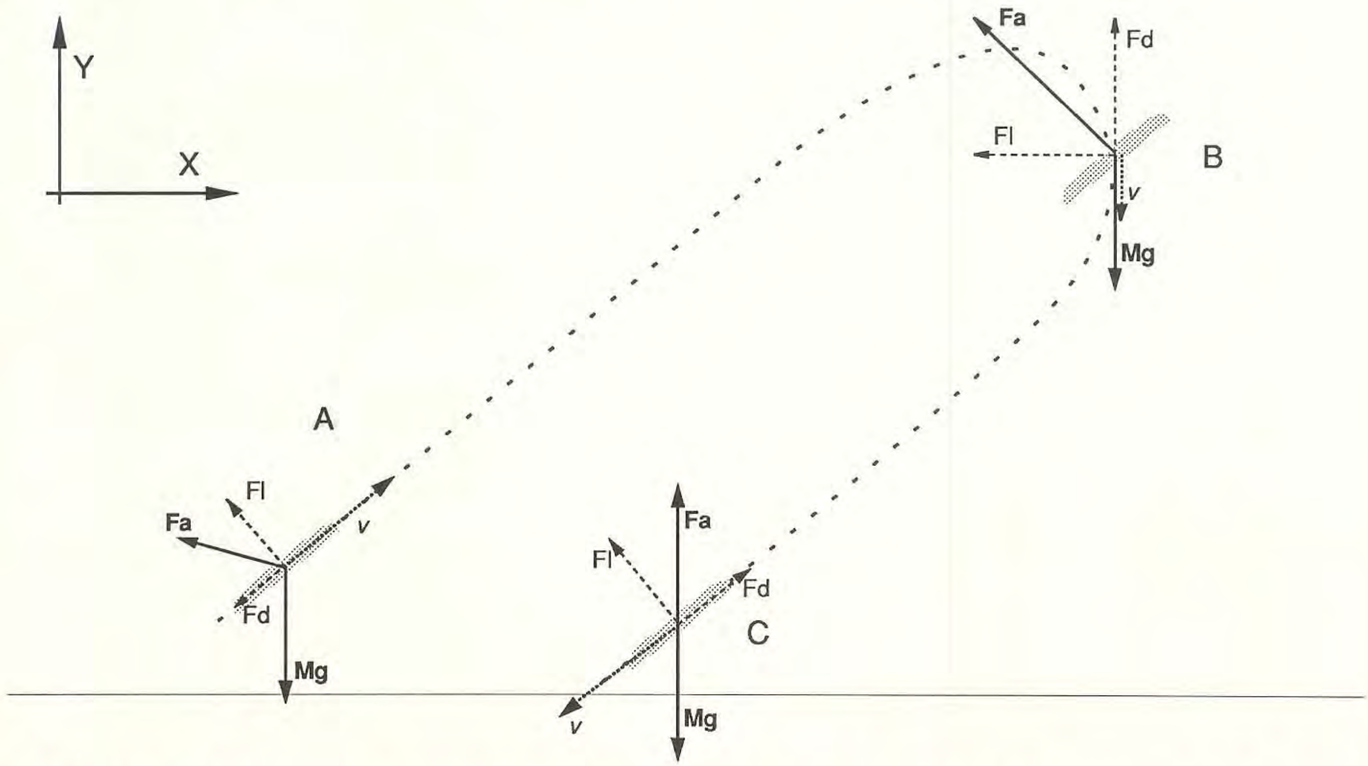

Fig. 13. A breakdown of aerodynamic and gravitational forces on the Frisbee during flight. A - right after launch. v=15 m/sec, α=0°. B - Free fall. v=7 m/sec, α=50°. C - Just before hitting ground. v=16.3 m/sec, α=-2°. Vectors are drawn to scale. β_0=40°.y_0, ρ, g, M and A are as in fig. 8.

The aerodynamic properties of any object are determined by its size and shape. However, there are several factors which affect the flight path of the Frisbee, or for that matter, any other object. In the above numerical analysis it was demonstrated how each of these factors: release angle (β_0), the object's mass (M), the release height (y_0) and velocity (v_0), the air density (ρ) and, to a much lesser extent, gravity (g), affect the flight of the Frisbee. The ability of the Frisbee to return, therefore, depends on the combination all of all these factors.

With a release angle above 0° (in fact above -24° where v_0 points to the ground), the combined aerodynamic force ($F_a = F_d + F_L$) tilts backwards and acts both as a slowing down force and as a counteraction to gravity. If the combination of the above factors is right, then the Frisbee's horizontal velocity (v_x) will reach zero and the Frisbee will be in a free fall situation. But not for long. The Frisbee's velocity, now only in the -Y direction, induces F_a which points backwards to a certain degree.

A Frisbee which began its flight with β_0=40° (see fig. 13) drops in its a/g ratio from the begining of 0.85 to a low of 0.19 at its maximal height (v_y=0). The ratio then rapidly increases to a high of 1.55 at the point were $v_x = 0$ and v_y is approximately 7 m/sec. This large aerodynamic force changes the direction of v quite rapidly until it points just about 180° to β_0. Although v increases and thus we would expect a further increase in a/g, the drop in α and thus a drop in C_d and C_L "compensates" and in fact reduces this ratio.

To help understand why v reaches a direction which is about 180° to β_0, and remains there, two cases will be examined. v' is the velocity which points β_0 -180°. In the first case the Frisbee flies at v' and α=0° and in the second v' is minus a few degrees and α is slightly negative. At α=0°, C_d =0.08 and C_L =0.12 and the ratio F_d / F_L =2/3 which results in F_a pointing backwards, roughly 56° from the plane of the Frisbee. At v=16.3 m/sec, ρ and M as in fig. 8, the ratio a/g equals 1.

With these parameters and at any angle β_0, the combined force (F_a + Mg) will point so its -X component will be larger than its -Y component. If v=16.3 m/sec and points exactly β_0 -180°, then the net acceleration will change v and make α negative. From a certain negative α and onwards F_a starts to point to the opposite direction. The new combined force acts now to increase α (make it less negative) and get back to the previous situation. What we have here is a self regulating system with a type of negative feedback control that acts to keep the Frisbee at an almost constant angle which is about β_0 - 180°. This equilibrium angle α_E is above all dependent on v itself. At v=16.3 m/sec I calculated using some linear interpolation, that α_E is between -1° and -2°. Since v increases all the time on the returning leg, α_E will gradually become a bit more negative.

ACKNOWLEDGMENTS

I owe many thanks to ms. Vardit Goldner who took part in conducting the experiments and to Prof. Raul Weil of the Technion for his support and enthusiasm. I would also like to thank the workers at the "Dantziger" workshop for their assistance with the operation of the wind tunnel and the crafting of the additional accessories. Eran Ron, Eitan Nachshoni and Adi Stern of Tel-Aviv University were kind enough to review an earlier draft of this manuscript and provide me with useful comments. Reuven Sherwin helped me with the graphics.

[1] D. J. Hall, S. L. Upton, A wind tunnel study of the particle collection efficiency of an inverted Frisbee used as a dust deposition gauge, Atmospheric Environment 22(7) Jul. 1988.
[2] F. Hess, The aerodynamics of boomerangs, Scientific American 219(5), Nov. 1968.
[3] C. Frohlich, Aerodynamic effects on discus flight. American J. of Physics 49(12) Dec. 1981.
[4] J. Walker, Boomerangs and More on Boomerangs, (In a Scientific American publication on rotational momentum, 1985).
[5] C. Proujan, The way it works, Science World 43(18) May 1987.
[6] G. T. Walker, On boomerangs, Philosophical Transactions of the Royal Society of London, Series A, Vol. 190, 1898.
[7] S. Johnson , *Frisbee* (Workman, New York, 1975).